Finite Field Harmonics for Stellarators with Improved Aspect Ratio

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To Professor Arnulf Schlüter on his 60th Birthday

A set of stellarator fields with magnetic surfaces of small aspect ratio is found by superposing recently introduced, easily computable field harmonics. The law of superposition is found analytically by closely approaching "helical symmetry" of these "new" fields in suitable orthogonal coordinates.

1. Introduction

As was pointed out in [1, 2], there exists a set of easily computable field harmonics which is especially suitable for representing vacuum magnetic fields in stellarators. This set has proved useful for the solution of 3D stellarator boundary value problems, the computation of continuous Rehker-Wobig coil systems [2, 3], which improved the intuitive background for designing "real", discrete coil systems, the optimization of stellarator fields, and the numerical analysis of equilibrium and stability properties [3, 4, 5]. One main requirement of the work in progress is to achieve field configurations with a magnetic well, a reduced ratio of $j_{\|}/j_{\perp}$ as compared with the standard $l=2$ stellarator and, simultaneously, a small aspect ratio of the outermost magnetic surface.

Field configurations with nested magnetic surfaces are obtained from the fields [1, 2] by superposing at least two "linear polarized" field harmonics with the same toroidal and poloidal periodicities to get the necessary "elliptically" or "circularly polarized" field. If this field is further "symmetrized" by a proper choice of the "poloidal harmonic content", i.e. by superposition of higher poloidal harmonics, then the aspect ratio of the outermost closed magnetic surface may be appreciably lowered if the comparison is made for the same rotational transform of this surface. This was found for fields with $l=1$ up to $l=4$ poloidal periods.

A symmetrization procedure with this effect will be described below. It yields a "new" set of fields by a linear transformation from the "old" set of "linearly polarized" elementary fields. The behavior of these "new" fields with respect to the toroidal coordinate is similar to the behavior of helically symmetric fields with respect to the coordinate along the straight axis. Each one of these fields of this "new" set has nested magnetic surfaces with a relatively small aspect ratio of the outermost regularly closed surface. With the "old" set of fields, comparable values of this aspect ratio for the same numbers of toroidal and poloidal field periods and for the same value of the rotational transform had only been obtained by a cumbersome optimization procedure or by the solution of a 3D boundary value problem [2].

2. Formalism

In Cartesian coordinates x, y, z the Laplace equation $\Delta U=0$ for the scalar potential U of the magnetic field is equivalent to

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-k^2\right)U=0 \tag{1}$$

if U is periodic in z corresponding to a linear combination of $\cos(kz)$ and $\sin(kz)$ with a constant-value of k. The introduction of polar coordinates $\bar{r}, \bar{\vartheta}$ by

$$x=\bar{r}\cdot\cos(\bar{\vartheta}), \quad y=\bar{r}\cdot\sin(\bar{\vartheta}) \tag{2}$$

changes (1) into

$$\left(\frac{\partial^2}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}-\frac{l^2}{\bar{r}^2}-k^2\right)U=0 \tag{3}$$

if U is periodic in z as well as in the poloidal angle $\bar{\vartheta}=\operatorname{arctg}(y/x)$ corresponding to a linear combination of $\cos(l\bar{\vartheta}-kz)$, $\sin(l\bar{\vartheta}-kz)$, $\cos(l\bar{\vartheta}+kz)$ and $\sin(l\bar{\vartheta}+kz)$. Here l is the number of field periods

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in the poloidal direction. Equation (3) thus allows partial solutions of U which are helically symmetric with respect to the coordinate z along the corresponding axis at $x = y = \bar{r} = 0$.

In cylindrical coordinates R, Φ, Z solutions of $\Delta U = 0$ may be constructed with a similar behavior with respect to the toroidal coordinate Φ along a circular "axis", e.g. at $R = 1$, $Z = 0$. This is true at least for small distances from this circle, i.e. for large aspect ratios with respect to $R = 1$, as will be shown.

Corresponding to [6], an orthogonal transformation from the cylindrical coordinates R, Z to new coordinates u, v by

$$u + iv = g(R + iZ) \tag{4}$$

transforms $\Delta U = 0$ into

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} - H(m^2 - \tfrac{1}{4})\right) R^{1/2} U = 0 \tag{5}$$

if U is periodic in Φ corresponding to a linear combination of $\cos(m\Phi)$ and $\sin(m\Phi)$. Here i is the imaginary unit, g is a function of the complex argument $R + iZ$, m is the number of toroidal field periods and H is defined by

$$H = 1/(R^2 g' \overline{g'}), \tag{6}$$

where (′) denotes the derivative with respect to the argument, and (—) the complex conjugation. The similarity between (5) and (1) shows that it is possible to obtain solutions of (5) for U or at least for the product $R^{1/2} U$ which are nearly helically symmetric in the frame of the coordinates u, v and Φ if $H \to 1$ for $R \to 1, Z \to 0$ can be approximated to a sufficiently high degree by a proper choice of the function g. A convenient choice is

$$g = \ln(R + iZ). \tag{7}$$

with (4, 6) this leads to

$$u = \tfrac{1}{2} \ln(R^2 + Z^2), \tag{8}$$

$$v = \operatorname{arctg}(Z/R), \tag{9}$$

$$H = 1 + (Z/R)^2. \tag{10}$$

Thus $H - 1 \ll 1$ is satisfied in second order with increasing aspect ratio. Different representations for g are possible of course. The order of the deviation of H from 1 cannot, however, be increased further if g is analytic in the region of interest near $R = 1$, $Z = 0$. The choice (7) for g should thus be sufficient.

In (5) we introduce a coordinate transformation similar to (3):

$$u = r\cos(\vartheta), \quad v = r\sin(\vartheta), \tag{11}$$

where r is a "distance" from the circular "axis" at $R = 1$, $Z = 0$, and $\vartheta = \operatorname{arctg}(v/u)$ is a "poloidal angle" in the frame of u and v. By inspection of (8, 9) it may be seen that, for large values of the aspect ratio, these two quantities indeed correspond to a distance and a poloidal angle with respect to the "axis" $R = 1, Z = 0$. We further introduce the quantity S, which shall be defined so that $U - S$ instead of U satisfies (5) if $H = 1$. It should be noted that U corresponds to a vacuum field but $U - S$ or S do not. By means of (11) an equation similar to (3) is obtained from (5):

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{l^2}{r^2} - m^2 + \frac{1}{4}\right) \times R^{1/2}(U - S) = 0 \tag{12}$$

if $R^{1/2}(U - S)$ is assumed to be doubly periodic with m periods in Φ and l periods in ϑ. With H given by (10) the equation for S may be written as

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} - H(m^2 - \tfrac{1}{4})\right) R^{1/2} S = -\frac{Z^2}{R^2}(m^2 - \tfrac{1}{4}) R^{1/2}(U - S). \tag{13}$$

Here $U - S$ is obtained from (12) as

$$U - S = I_l((m^2 - \tfrac{1}{4})^{1/2} r) R^{-1/2} F(\Phi, \vartheta), \tag{14}$$

where F is a linear combination of $\cos(l\vartheta - m\Phi)$, $\sin(l\vartheta - m\Phi)$, $\cos(l\vartheta + m\Phi)$ and $\sin(l\vartheta + m\Phi)$, and I_l denotes the modified Bessel function of order l. This yields the required, nearly "helical symmetric" solution for the scalar potential U of a typical stellarator field with m toroidal and l poloidal field periods at least for large values of the aspect ratio (where $R \to 1$), if S becomes small in this case. To get the required property of S, it shall be subject to the following Cauchy boundary conditions:

$$S = 0, \quad \partial S/\partial R = 0, \quad \text{for} \quad R = 1. \tag{15}$$

This yields

$$U = U - S, \tag{16}$$

$$\frac{\partial U}{\partial R} = \frac{\partial(U - S)}{\partial R}, \quad \text{for } R = 1, \tag{17}$$

where $U - S$ is the expression on the right-hand side of (14). At large values of the aspect ratio the right side of (13) together with (15) guarantee S to be smaller than $U - S$ by at least two orders in the aspect ratio. With a corresponding choice of F in (14) a good approximation of "helical symmetry" by $R^{1/2} U$ in the above stated sense can therefore be expected.

The last remaining step is to represent U by linear superposition of our field harmonics defined previously in [1, 2]. The most convenient way towards establishing this connection is found by comparing the Cauchy boundary conditions to be satisfied by U at the cylindrical surface $R = 1$ as stated by (16), (17) with the Cauchy boundary conditions which are satisfied at the same surface by our fields as stated in [1] (loc. cit. Eqs. (19), (20) or in [2] (loc. cit. Eqs. (11)—(14). Since a vacuum field is uniquely determined by Cauchy conditions on a piece of a surface, the coefficients of a linear combination of our fields which has to represent U are thus uniquely determined by (16), (17). The practical evaluation is done by first equating Fourier coefficients in (16), (17) which belong to the same factor $\exp(im\Phi)$. For these Fourier coefficients a representation in powers of the small quantities $\ln(R)$ and Z is used. The coefficients of the linear combination are then obtained by $R -> 1$ and by equating the remaining sub-components belonging to the same powers of Z in (16), (17).

3. Analytical Results

The analytical evaluations were made by means of REDUCE for $l < 5$ in (14) and for arbitrary values of m. The explicit results are represented in the Appendix and are summarized here.

For the present description of the "new" fields in terms of the "old" ones we use the notation of [2] for the "old" quantities. This especially concerns the symbols $D_{m,l}$ and $N_{m,l-1}$, which were used to denote the "Dirichlet" and "Neumann" type Fourier coefficient of the scalar potential of an "old" elementary field with m toroidal and l poloidal field periods (see Eq. (10) in [2]).

The essential result of the present considerations is formally simple. It merely states that the "old" expressions $D_{m,l}$ and $N_{m,l-1}$ are replaced by corresponding "new" expressions $D^*_{m,l}$ and $N^*_{m,l-1}$ which can be explicitly represented by the "old" expressions, as shown in the Appendix. The qualitative physical meaning of m and l in the "new" expressions is exactly the same as for the corresponding "old" ones. Apart from some differences, the "new" fields could therefore be used in the "old" context as well. There is merely a quantitative difference of higher order in powers of Z and $\ln(R)$ or $R - 1$ which tends to "symmetrize" the "new" elementary fields with respect to the poloidal variation of the modulus of their poloidal field component. Closely connected to this is another difference noted at the end of this section. The main difference lies in the fact that the "new" elementary fields have the desired "poloidal harmonic content" as mentioned in the introduction, which leads to elementary magnetic fields with approximate "helical symmetry" and to magnetic surfaces of small aspect ratio. The corresponding scalar potentials $V^*_{m,l} = V^*_{m,l}(R, \Phi, Z)$ of these "new" elementary fields are given by

$$V^*_{m,l} = \begin{cases} D^*_{m,0} \sin(m\Phi), & l = 0, \\ D^*_{m,l} \cos(m\Phi) \mp N^*_{m,l-1} \sin(m\Phi), & l = 1, 3, \ldots, \\ D^*_{m,l} \sin(m\Phi) \pm N^*_{m,l-1} \cos(m\Phi), & l = 2, 4, \ldots. \end{cases} \tag{18}$$

where m is non-negative. The upper sign corresponds to a left-hand screw or polarization, the lower sign to right-hand polarization. The case $l = 0$ corresponds to something like a periodic cusp field which is nearly "axially symmetric" with respect to the "axis" Φ. Together with the scalar potential $V = \Phi$ of the main field it causes a periodic variation of this field and of the cross-section of corresponding flux tubes. The symmetry of (18) is such that

$$V^*_{m,l}(R, -\Phi, -Z) = -V^*_{m,l}(R, \Phi, Z). \ldots$$

It corresponds to a convention (denoted as "case S3" in [2]) where the cross-section of the magnetic surfaces in the meridional plane $\Phi = 0$ is symmetric with respect to an interchange of the sign of Z. The other type of symmetry where

$$V^*_{m,l}(R, -\Phi, -Z) = +V^*_{m,l}(R, \Phi, Z)$$

(denoted as "case S2" in [2]) could be obtained by an interchange of $m\Phi$ with $m\Phi + \pi/2$ in (18).

The left-hand and right-hand polarized versions of (18) together form a set which is complete within the above-mentioned symmetry "case S3", and which has the above-mentioned property regarding

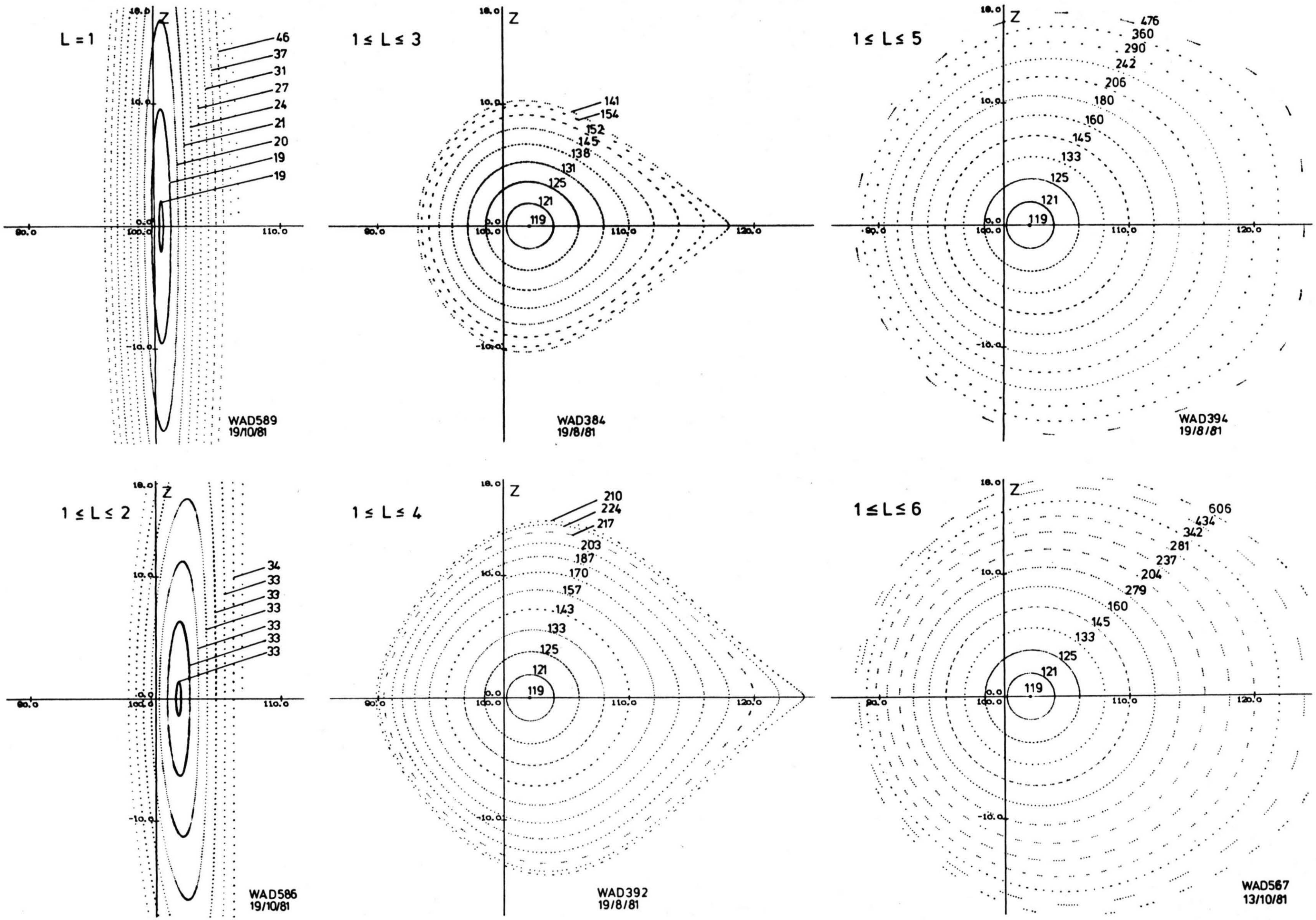
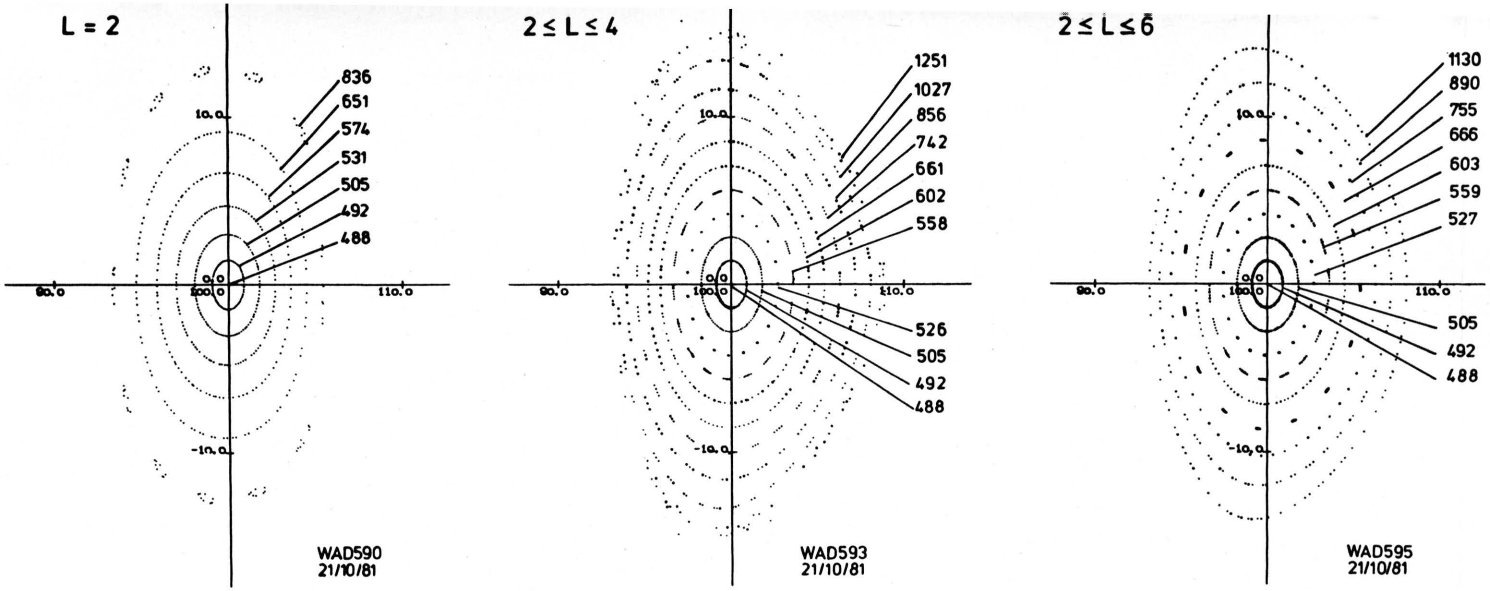


Fig. 1. Cross-sections of magnetic surfaces in the meridian plane $\Phi = 0$ of different fields (see text) with $m = 10$ toroidal and $l = 1$ poloidal field periods. The case denoted by $L = l$ corresponds to an "old" field (in "circularly polarized" form). In cases denoted by $l \leq L \leq L_{\text{max}}$ the harmonic content of the "old" field is changed according to our results for harmonic numbers $L > l$ in the indicated range. These cases demonstrate successively improved approximations to the corresponding "new" $l = 1$ field up to $L_{\text{max}} = 6$. Numbers at the surfaces give the rotational transform times 1000.



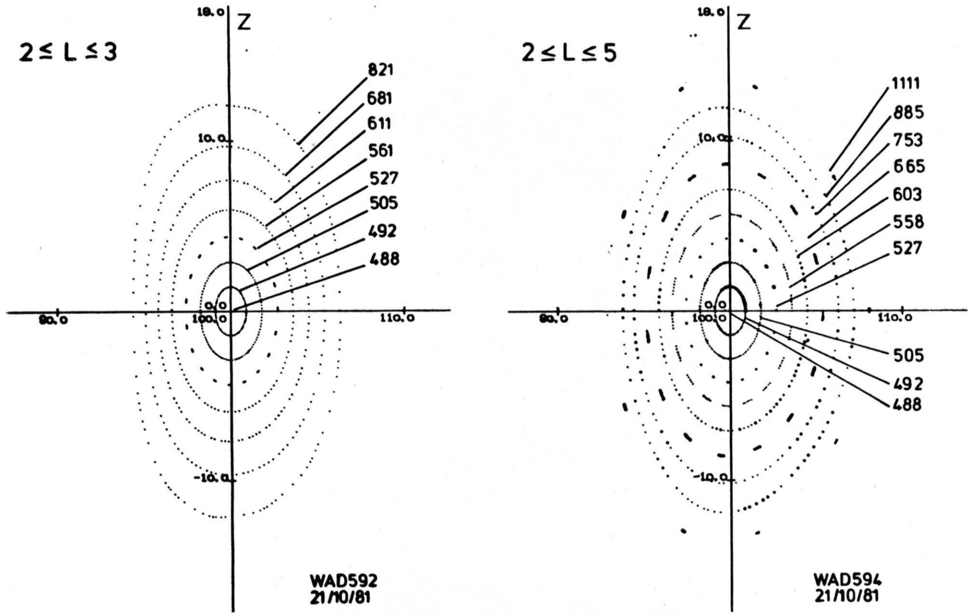


Fig. 2. An example similar to Fig. 1 is shown here for $l = 2$.

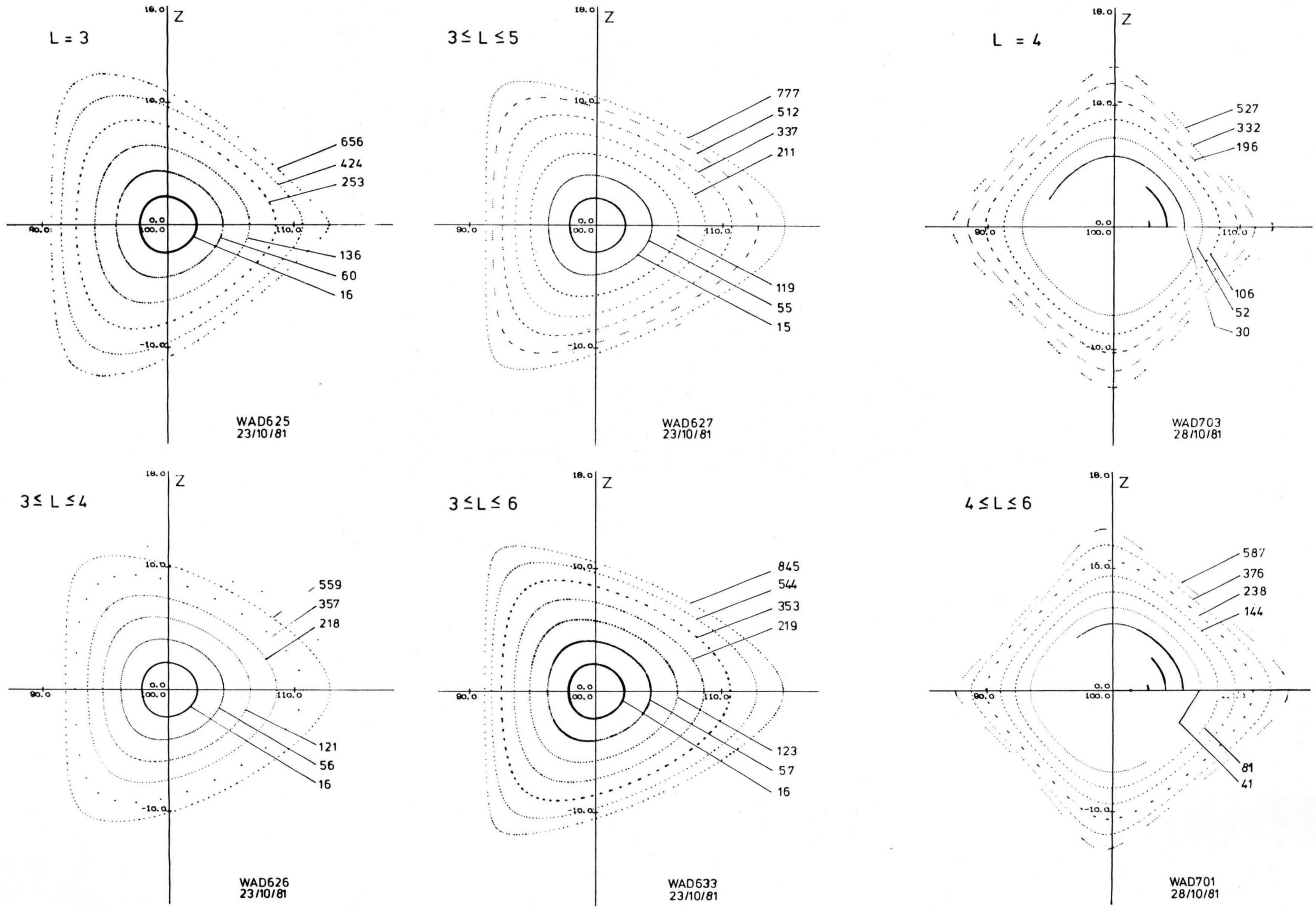


Fig. 3. An example similar to Fig. 1 is shown here for $l=3$ and $l=4$.

the magnetic surfaces. If the last property is not needed directly, a complete set with the same symmetry can be obtained of course by using the "Dirichlet" and "Neumann" terms in (18) as separate potential functions. It should be noted, however, that these "new" terms do not satisfy exact Dirichlet or Neumann conditions at $R=1$, as was the case with the corresponding "old" ones. The "new" terms only possess this quality in lowest order in Z, as may be seen from the structure of the expressions given in the Appendix.

4. Numerical Results

Numerical tests of the analytical results of Sect. 3 were performed by computing corresponding magnetic surfaces. Sets of such surfaces will be shown in Figs. 1 to 3 in their meridional cross-section at $\Phi=0$ for typical stellarator fields with $m=10$ toroidal and $l=1, 2, 3, 4$ poloidal field periods whose scalar potential is given by $\Phi + c.\, V^*_{m,l}$. The toroidal angle Φ represents the normalized scalar potential of the main field in this expression, c is a constant factor and $V^*_{m,l}$ is obtained from (18). This is shown for $l=1$, $c=0.25$ in Fig. 1, $l=2$, $c=2.155$ in Fig. 2, $l=3$, $c=25.0$ and $l=4$, $c=500.0$ in Figure 3. Here 100 means 1 for the coordinates and 1000 means 1 for the indicated values of the rotational transform (defined as rational for closed field lines). Up to 6 different sets of nested surfaces are shown for each value of l. As is indicated in the figures by $l \leqq L \leqq L_{\max}$, these sets differ from each other in their content of poloidal harmonics in the expressions given in the Appendix (where the harmonics are numbered by L). Sets indicated by $L=l$ have $L_{\max}=l$ and correspond to "old" fields (in "circularly polarized" form). The other sets demonstrate successive approximations to the corresponding "new" field where the poloidal harmonic content of the old "field" has been changed according to our results for $L>l$ up to increasing values of $L_{\max}$.

As may be seen, the aspect ratio of the outermost closed magnetic surface is lowered and its rotational transform is increased simultaneously if for given numbers m and l and with a constant value of c an "old" field is replaced by some higher order approximation of the corresponding "new" one. This of course means an even larger reduction of the "new" aspect ratio if the comparison is made at a constant value of the "old" rotational transform of the outermost closed surfaces rather than at a constant value of c. Improvements of the aspect ratio by factors between 1.6 and 2.2 were found in this case for $L_{\max}=6$ for the examples given and others not shown here. One of the latter with $m=19$, $l=2$ and a rotational transform of 0.52 at the magnetic axis ($c=3.1$) approximately corresponds to conditions in the Heliotron E experiment [7]. The outermost magnetic surface of a "new" $l=2$ field has a rotational transform of 2.3 and an aspect ratio of about 13 in this case, whereas about 26 was obtained with an "old" $l=2$ field with the same rotational transform at the outermost magnetic surface, where the value on the magnetic axis had to be changed to 1.5 ($c=5.1$). Conditions in the Heliotron E correspond to a rotational transform of about 2.5 at an aspect ratio of 10 in this case. The conditions reached with the "new" $l=2$ field thus do not fall short much of actual conditions though no attempt was made to reduce the aspect ratio further by an admixture of other field harmonics. This points to good convergence with respect to practical applications.

Of special interest in our case is the optimization of stellarator fields [3]. Each computer run needs many field evaluations and test displacements in the parameter space of the coefficients of linear combinations of fields. The "new" fields have thereby proved superior to the "old" ones since the parameter space is compacted to smaller values of l with respect to the property of small aspect ratio even in the case of a combination of "new" fields with different helicity. This reduces the dimensionality of the parameter space required for the optimization and largely excludes unwanted degrees of freedom which would allow for ill-conditioned configurations.

Meridional cross-sections of magnetic surfaces at $m\Phi/2\pi = 0, 1/4, 1/2$ of an optimized $m=5$ configuration are shown as an example in Figure 4. It was obtained from a mixture of "new" fields with $m=5, 10$ and $l=0, 1, 2, 3$ periods by requiring simultaneously a magnetic well and a small poloidal B-variation along equipotential lines (to reduce $j_\parallel/j_\perp$ and the particle drift component normal to the magnetic surfaces) at one of the outermost magnetic surfaces with an aspect ratio of about 8 at reasonable form parameters. The resulting compromise shown here has a rotational transform of 0.32 at the magnetic axis and about 0.5 at this surface, where a magnetic well of 2.3 per cent is maintained and the poloidal B-variation is 80 per cent (and less inside this surface) as compared with an $l=2$ stellarator with the same rotational transform and the same aspect ratio. A reduction of transport in the regime of small collisionality to less than 50 per cent of the $l=2$ stellarator was found in this case by Lotz [8] with his Monte Carlo code for neoclassical transport [9].

5. Conclusion

A set of "circularly polarized" stellarator fields with small aspect ratio of the corresponding magnetic surfaces was constructed by suitable superposition of recently introduced, easily computable toroidal field harmonics [1, 2]. The law of superposition was found analytically from the requirement that each of these "new" elementary fields approximate "helical symmetry" around the circular toroidal "axis". The "new" set, tough equivalent to the "old" one in principle, has proved superior in optimizing stellarator fields. The reason is the small aspect ratio of the magnetic surfaces of the "new" elementary field harmonics, which has mostly proved to remain small even for linear combinations of different "new" harmonics of low order.

Acknowledgement

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Appendix

The following shows the explicit representation of the "new" functions $D^*_{m,n}$ and $N^*_{m,n}$ by the "old" functions $D_{m,n}$ and $N_{m,n}$ as explained in Section 3. Columns IDN and L at tight give FORTRAN code numbers of the affected "old" fields in the currently used field configuration data sets. The corresponding coefficients are identical with the coefficients (including the sign at left) which are associated here with $D_{m,n}$ or $N_{m,n}$.

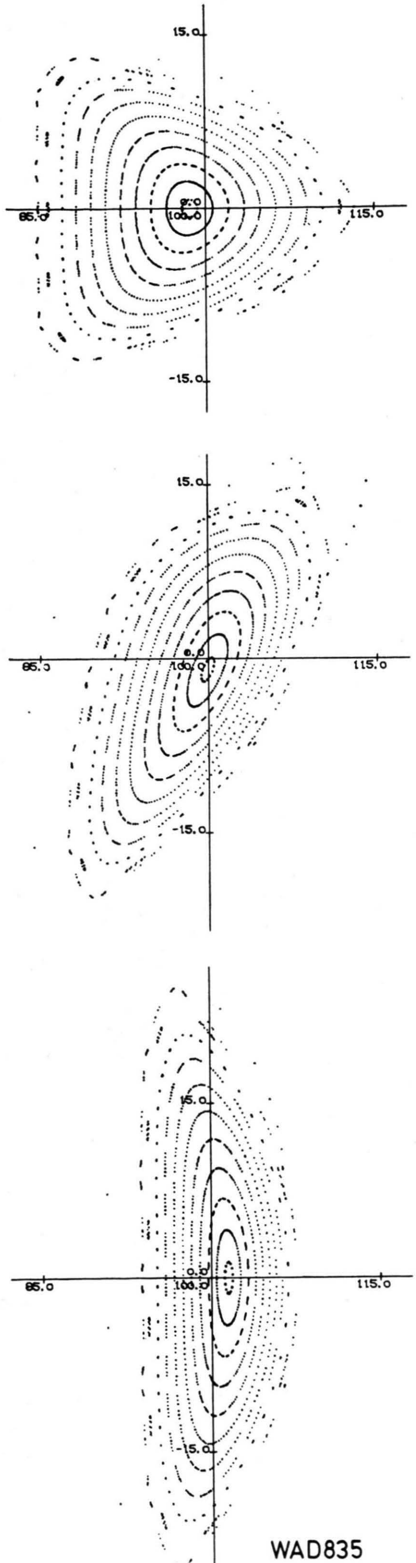

Fig. 4. Meridional cross-sections (at $m\Phi/2\pi = 0, 1/4, 1/2$) of magnetic surfaces of a field configuration with $m=5$ periods obtained by optimization of a mixture of "new" fields with $m=5,10$ and $l=0, 1, 2, 3$ periods. Further details are given in the text.

		IDN,	L
$D^*_{m,0} =$	$D_{m,0}$	1	0
	$- N_{m,0} \cdot 1/2$	2	1
	$+ D_{m,2} \cdot (4\,m^2 - 1)/8$	1	2
	$- N_{m,2} \cdot 3\,(4\,m^2 - 1)/16$	2	3
	$+ D_{m,4} \cdot (48\,m^4 - 344\,m^2 + 83)/128$	1	4
	$- N_{m,4} \cdot 3\,(80\,m^4 - 744\,m^2 + 181)/256$	2	5
	$+ D_{m,6} \cdot (320\,m^6 - 9840\,m^4$		
	$+ 52988\,m^2 - 12637)/1024$	1	6
	....		
$D^*_{m,1} =$	$D_{m,1}$	1	1
	$- N_{m,1} \cdot 3/2$	2	2
	$+ D_{m,3} \cdot (12\,m^2 - 35)/16$	1	3
	$- N_{m,3} \cdot (60\,m^2 - 239)/32$	2	4
	$+ D_{m,5} \cdot (80\,m^4 - 1480\,m^2$		
	$+ 3437)/128$	1	5
	$- N_{m,5} \cdot (560\,m^4 - 12920\,m^2$		
	$+ 36987)/256$	2	6
	....		
$N^*_{m,0} =$	$N_{m,0}$	2	1
	$+ D_{m,2}$	1	2
	$+ N_{m,2} \cdot (4\,m^2 - 41)/16$	2	3
	$+ D_{m,4} \cdot 3\,(4\,m^2 - 17)/8$	1	4
	$+ N_{m,4} \cdot (16\,m^4 - 840\,m^2$		
	$+ 3665)/128$	2	5
	$+ D_{m,6} \cdot 15\,(16\,m^4 - 360\,m^2$		
	$+ 1113)/128$	1	6
	....		
$D^*_{m,2} =$	$D_{m,2}$	1	2
	$- N_{m,2} \cdot 7/2$	2	3
	$+ D_{m,4} \cdot (4\,m^2 - 45)/4$	1	4

		IDN,	L
	$- N_{m,4} \cdot 3\,(12\,m^2 - 151)/8$	2	5
	$+ D_{m,6} \cdot (240\,m^4 - 10360\,m^2$		
	$+ 72719)/256$	1	6
	....		
$N^*_{m,1} =$	$N_{m,1}$	2	2
	$+ D_{m,3} \cdot 3$	1	3
	$+ N_{m,3} \cdot (4\,m^2 - 101)/8$	2	4
	$+ D_{m,5} \cdot 5\,(4\,m^2 - 41)/4$	1	5
	$+ N_{m,5} \cdot (80\,m^4 - 7720\,m^2$		
	$+ 78469)/256$	2	6
	....		
$D^*_{m,3} =$	$D_{m,3}$	1	3
	$- N_{m,3} \cdot 13/2$	2	4
	$+ D_{m,5} \cdot 5\,(4\,m^2 - 113)/16$	1	5
	$- N_{m,5} \cdot 5\,(60\,m^2 - 1567)/32$	2	6
	....		
$N^*_{m,2} =$	$N_{m,2}$	2	3
	$+ D_{m,4} \cdot 6$	1	4
	$+ N_{m,4} \cdot (12\,m^2 - 611)/16$	2	5
	$+ D_{m,6} \cdot 45\,(4\,m^2 - 81)/16$	1	6
	....		
$D^*_{m,4} =$	$D_{m,4}$	1	4
	$- N_{m,4} \cdot 21/2$	2	5
	$+ D_{m,6} \cdot (12\,m^2 - 683)/8$	1	6
	....		
$N^*_{m,3} =$	$N_{m,3}$	2	4
	$+ D_{m,5} \cdot 10$	1	5
	$+ N_{m,5} \cdot (4\,m^2 - 361)/4$	2	6
	....		

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